MATHEMATICS FOR ALL AND THE DOUBLE TRACK PLAN*

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In approaching this subject, I wish to be thought of not as an educationist nor as a former teacher of mathematics alone, but as one who has for a quarter century been active and maintained a keen interest in both areas. In discussing the future of mathematics in the high schools, two kinds of approaches have been commonly employed—one of these has been far more popular and in all likelihood less useful. It is what may be called the "Pollyanna" or "rabble rouser" approach. The author or speaker makes the strongest case he can for teaching mathematics in the schools and refuses to recognize any of the dangers or any of the needs for improvement or reorganization of the content. The second approach is much more hazardous, though probably much more serviceable. It may be called the "Prophet Jeremiah" or "Dutch Uncle" approach. In this approach the speaker or author takes for granted that mathematics should be taught to more and more people. He addresses himself to the dangers in the present situation and dwells greatly upon the weaknesses and upon the possibilities of improvement of the present offerings. I would have very little interest in approaching this discussion along the first of these two lines.

I wish to make two principal points:

1. Everyone should study mathematics in high school and should have at least a year of mathematics beyond the tenth grade, and in all, not less than two years of high school mathematics.

2. The conditions of the times clearly call for two sequences of mathematics in the high school (a) one for the growing mass of high school students, a sequence to give them the mathematical information, skills and attitudes necessary to meet problems of everyday life in home, business, recreation, vocation, citizenship and health and (b) a sequence for the specialist—the student of more than average ability in mathematics—the future engineer or college student in some phase of physical science.

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* An address to the high school section of teachers of Mathematics of the Central Association of Teachers of Science and Mathematics, Chicago, Nov. 28, 1944.

One Thing We Learned from the War

From our experiences during the war we have been stimulated to think more carefully about the need for training in mathematics. I served as a member of the committee to select students for the Navy V-12 program and from that experience learned a great deal about the needs of the Navy and about the background of young people today in mathematics. I should like first of all to call your attention to the fact that the officers of the Navy who wrote or spoke on these matters emphasized:

1. that a considerable minority of young men entering the Navy should be well trained in mathematics beyond algebra and geometry, and

2. that the great majority should be thoroughly trained in arithmetic, and elementary algebra and preferably also in elementary trigonometry. In the refresher or pre-induction course of study in mathematics planned by the Navy it is recommended that *40% of the time be given to reviewing and extending skills in arithmetic.*

How Much Arithmetic in the High School Mathematics Courses?

Perhaps those of us who are inclined not to take seriously the suggestion that considerable attention be given to arithmetic in the high school should remember that at one time arithmetic was taught at Harvard University and other leading institutions of higher education, and that for a long time it was a most respectable secondary school subject being required for entrance to colleges and universities. It is easy to understand the pride of one, who having majored in mathematics in college, and how that pride might mislead one to the conclusion that he or she is above teaching arithmetic, particularly when one knows what difficulty many mathematics majors have in solving many of the problems in arithmetic and how lacking in background they are with respect to the areas of application of arithmetic. It is also not impossible to understand how that pride may stand in the way of the welfare of mathematics and of secondary education.

There are several reasons, however, why we must at this time consider the matter more carefully. In the first place there is a growing and articulate criticism of the graduates of our high schools because of their very inadequate mastery of arithmetic. In addition, the postponement in grade placement of more diffi-

cult topics in arithmetic in the elementary school has made it impossible for elementary school teachers to complete the study of arithmetic before the ninth grade. It is futile to suspect that this postponement policy will be reversed. Teachers in the elementary schools have come to believe fervently in the futility of attempting to crowd learning of mathematics too rapidly. On every hand, we are discovering so many individuals who not only did not master arithmetic in the elementary school but who as a result of this crowding process have developed antipathy towards mathematics and a lack of confidence in themselves which is almost psychopathic.

If this postponement movement should result in deferment of systematic instruction in formal algebra to the tenth grade, then we must consider favorably beginning of algebra at that time and the teaching of a great deal of arithmetic in the senior high school. Such a plan of postponement would not be without important advantages for instruction in algebra, begun at a later age and taught to a more select group who would have less time to forget it before entering college. The way of the algebra teacher of today, like that of the transgressor, is hard, expending as he or she does, major efforts and worries in a futile attempt to make mathematical silk (rayon) purses out of sows' ears, to the shameful neglect of the able student.

More Students or Fewer to Study Mathematics in High School?

The present increasing popular interest in mathematics is likely to be ephemeral. The public is fickle and its interests shift quickly. We must "strike while the iron is hot." We should keep our customers coming by giving them the type of training most valuable to them and most completely in harmony with the needs of the time and the future.

Enrollments in mathematics in the high school school have diminished steadily for forty years, most probably because our courses, as offered, have not been suited to the needs of the masses in modern times. We have in our actions said, "Here is our course for the specialist. Take it or leave it. If you have no practical need for it be satisfied that it has disciplinary and general mental training." Below are statistics demonstrating the failure of that policy to be convincing as indicated by the gradual disappearance of high school mathematics from the programs of the majority of high school students. The figures in the state-

ments are percentages of all students in the high school enrolled in mathematics classes:

	All Algebra	Plane and Solid Geom.	All Math.
1900	56.3	27	91
1922	40	23	64
1928	35	19	57
1934	25	13	44
1940 (Estimated)	22	10	36

	Elem. Algebra	Plane Geom.
1928	27	18
1934	19	12
1940 (Estimated)	15	9

We should bear in mind that these decreases have taken place in spite of the continued requirement of mathematics for college entrance—a crutch which has, I fear, caused us to take it too easy and to worry too little about the future of secondary school mathematics. There are indications that the prop of college entrance requirements may be gradually pulled from under us. Already there is a movement under way to discontinue the requirement of mathematics for general college entrance. Not only do many teachers colleges no longer have such a specific requirement but entrance requirements have been revised so that students may enter without credit in either algebra or geometry, in a great many colleges and in a number of our larger universities including such institutions as the University of Michigan, University of Iowa, University of Wisconsin, University of Illinois and Stanford University. Repeated careful investigations have shown that in colleges of arts and science, schools of music and schools of business administration, students who have not presented, for entrance, credits in high school mathematics make as high scholastic average as do students of the same general intelligence who have had two or more years of high school mathematics.

Everyday Needs Becoming More and More Mathematical

It defies explanation and is contrary to all logic that as the world becomes more mathematical, fewer students in high school take courses in mathematics. There is a screw loose somewhere—an Ethiopian in the proverbial woodpile. If time were available and if it were not carrying "coals to Newcastle," I would like to help you picture in your minds this afternoon just

how mathematical a world we have become and just how much more mathematical we are becoming, how almost every hour in every day a great majority of us have some thought partaking of a quantitative nature if not actually involving some computation and mathematical problem. I want to point out just a few of the elements in this trend:

1. The home of today is no longer a place of self-sufficiency. It has become and is becoming more and more a place of purchasing rather than producing. Even in our processing we have become more mathematical.

2. In the home of today financial matters have become more definitely mathematical as exemplified by our greater participation in insurance, annuities, budgets, to say nothing of the mathematics involved in all of the various scientific gadgets which make work easier in the home, or the mathematics involved in home decoration, or the mathematics relative to installment buying, borrowing, increased variety, amount and complexity of taxes.

3. Matters of health have become mathematical, what with our thinking in terms of calories, weights, etc.

4. Our methods of transportation not only are employed much more frequently but involve much more mathematical thinking, particularly as a result of the replacement of horse power in the flesh by horse-power in the combustion engine, with all its gauges and instruments and related calculations. We never thought in terms of how many miles to the bale of hay or bushel of corn, but we must think in terms of how many miles to the gallon. We must think also in terms of transportation expenses on the bus, in the airplane, by trucks, parcel post, etc., and be able to compute the costs of owning and operating an automobile.

5. In our plan of vocational activities, measurements play a much more prominent part both in original construction and in repair. This is particularly true in those vocations having to do with building construction and with gasoline engines and related machines of one kind or another.

6. To be an intelligent citizen in these days one must do considerable thinking in mathematical terms—in terms of magnitude and ratios—about the implications of national expenditures and budgets far beyond the capacity of the untrained mind to comprehend.

As Fibber McGee says these days "Pardon the expression"

but what a cockeyed world this is in which which mathematics plays an ever increasing part and mathematics education is constantly diminishing. At this point, let us consider what may be done about the matter. As possibilities I wish to present some proposals which no doubt would be modified this way or that if adopted in individual schools and which should go far to bridge the gap between the needs of today and our traditional school program.

Track One of a Double Track Plan—For the Masses

I should like to have you think about a double track plan—one for the masses and one for the specialist. Track one might have several forms. For example it might well take the form of requiring a semester of mathematics and perhaps a semester of science in every year through the high school, thus assuring that mathematics is not dismissed from the thinking of any student at the end of the eighth grade or at the end of the tenth grade and that he may grow up in an atmosphere which is somewhat mathematical and have his instruction developed along with his maturity and his expanding experience and interests.

This required science and mathematics would naturally involve much arithmetic and many of its difficult and complicated applications to the problems of life in all its various aspects. It would involve a great deal of what is now thought of as first year algebra, emphasizing as it would thorough training in the use of formula and the simple equation, graphical representation and negative quantities.

An alternative plan for Track One would involve a required year of general mathematics either in the ninth or tenth grade and again another required year in the eleventh or twelfth grade. This plan has one practical advantage in that there are textbooks already available for general mathematics in grades nine and ten and in general mathematics in grades eleven and twelve.

I am not inclined to take very seriously the claims of those who believe that we should have specialized courses in mathematics such as shop mathematics, business arithmetic, or agricultural mathematics. It is not possible to offer several such courses in small schools and the Track One general courses should serve almost equally well for vocational purposes and much better for various aspects of general education.

"Senior General Mathematics"

Already a rapidly increasing and very considerable number of schools are offering the course in *Senior Mathematics*, some only as a refresher course, but an increasing number as something more than that—a course in the skills and applications of arithmetic, simple algebra, geometry without proofs, and simple trigonometry. In a considerable number of schools, there is taught the use of the slide rule and of logarithms on both tracks but particularly on Track One. We should give considerable attention to types of outcomes other than mere calculation outcomes involving such as (a) ability to read carefully and think logically about mathematical problems (b) certain attitudes and habits of mind such as critical attitude relative to proof, habits of thinking about cause and effect and other interrelationships, (c) importance of developing a wholesome attitude toward mathematics and of avoiding an unhealthy attitude towards it or toward the school as a result of unfortunate experiences in mathematics classes. To those who would say that these types of outcomes smack of mental discipline, it may be said that indeed they do—of discipline of a practical and attainable nature and that such discipline should also include such things as habits of checking, the use of approximate estimation and rough or rounded calculations.

In Track One, much more attention should be given to *mental* calculations, realizing that in a fairly considerable percentage of our needs for mathematics, we must make our decisions without the use of pencil and paper even though our calculations are only approximate.

Track Two for the Specialist

Track Two is a familiar one to most of us. It is a well-known track, one which has been employed for generations. It is the track for the specialist, the prospective engineer, scientist, or teacher of mathematics, and for those who wish to enter a college or university which still requires formal algebra and deductive geometry. This well-worn track is in need of straightening on the curves, new ballast, and streamlining. In most schools it probably should begin in the tenth grade with a thorough one-year course in formal algebra. In the eleventh grade, plane and solid geometry would constitute the work of the year and in the twelfth grade more algebra, more arithmetic and ele-

mentary trigonometry, the time being divided approximately equally.

In larger schools this track might well begin with a year's course in algebra in the ninth grade, with plane geometry in the tenth grade, algebra and trigonometry in the eleventh grade, and algebra and solid geometry in the twelfth. In such schools, however, admissions to the algebra class in the ninth grade should be given freely only to the student who is at least in the upper half of the class with respect to ability to do mathematics, probably only to those in the upper one-third. Those in the middle third might well be advised to postpone algebra until the tenth grade, and those in the lower third encouraged to avoid disaster. The teaching of plane and solid geometry in one year need not be apologized for. In the smaller schools students wishing to take solid geometry are not numerous enough to warrant giving it. In any school devoting one and one-half years to instruction in geometry there is a disproportionate emphasis on geometry as compared to algebra in view of its practical advantages, even may I say for those who will be engineers or scientists.

In those schools in which a rigorous instruction in formal algebra is begun not before the tenth grade, a course in general mathematics should probably be offered and might well be common to both Track One and Track Two. It might be, however that where this plan is followed, two kinds of general mathematics should be offered: one for the students of lower ability and one for the students of greater ability, as far as mathematics is concerned, and the courses of study appropriately differentiated. In any case, the course in general mathematics in the ninth grade should not be, as it is in some schools, a diluted or camouflaged course in albegra. It should contain considerable arithmetic, some algebra, and without question, some geometry, the algebra emphasizing skill in the use of formulas, simple equations, and graphs, and the geometry emphasizing construction scale drawings, and indirect measurement.

Will the Double Track Plan Become Widespread?

To sum up thus far, the Track Two kind of mathematics, important as it is, is not suited to the needs of the majority of high school students. We can no longer stave off the trend against us or dodge our responsibility with vague phrases about mental discipline and training the mind. Society demands that,

but also more. Not only do the needs of those in the armed forces, but also the needs of the average person, call for a disciplined mind with specific abilities to meet specific situations calling for mathematical thinking. Disciplinary values only are not enough. People are beginning to feel rather generally that other subjects probably have equivalent disciplinary values.

Not only will the percentage of students in high school studying the Track Two variety of mathematics continue to decrease but just prior to the war, in several states, perhaps many, the actual numbers decreased, New York and California for example. Even if we are not interested in numbers, we can hardly turn a deaf ear to the demands of the times and what is more to the point administrators looking for teachers of mathematics will find that after the war those who can teach Track Two courses will be very plentiful and easy to get. Those who can teach Track One courses will not be sufficiently numerous to meet the demand and will therefore be most sought after.

What stands in the way of our making the adjustment that seems indicated by recent developments and current needs. It is interesting to survey these:

1. Just plain tradition, inertia—lead in our mental boots.
2. The present abilities and attitudes of teachers. Many prefer the old way, the easy, simple way of teaching the regular mathematics out of the textbook, justifying our procedure and quieting our consciences with vague and wild theories about discipline and college preparation.
3. Many teachers are creatures of the classroom, who know little of the application of mathematics to life and are not at home in discussing such things with youngsters.
4. Many teachers either do not have, or fear they do not have, sufficient imagination, ingenuity and scholarship to organize the new courses and to teach them.
5. The conservatism of parents who apparently would like to keep the schools just as they were when they attended them and who are befuddled by theories of general mental training.
6. A considerable but decreasing number of administrators who prefer to be among the last to take up the new.
7. Suitable textbooks have not been available. This limitation is rapidly disintegrating. We now have several excellent three book series in junior high school mathematics enabling us to give an integrated sequence through the ninth grade, previous to the study of formal algebra. Within the past year several excellent books for senior mathematics have been published. Already a few textbooks combining plane and solid geometry in a one year course, have made their appearance and no doubt more such books will appear.
8. Stubbornly decreasing ignorance and misinformation about how to prepare students for college and a reliance upon the taking of certain subjects rather than upon the methods proven to be more effective in preparing students to do college work.
9. The mistaken, though genuine and sincere, attitude of a considerable

number of college professors of mathematics, who have had no experience in thinking in terms of mathematical needs of other than engineering students and majors in mathematics and physics. These men and women usually are quite unfamiliar with the high school population of today and have given little careful thought to the problem of what really are their mathematical needs. I am not one of those who feel that the college professor of mathematics is motivated largely by fear that if the newer trends in high school mathematics are adopted, teachers will have little need for courses in analytical geometry, calculus and differential equations.

Suggestions for Improving Teaching of Mathematics

I would like to conclude with a few statements about the nature of instruction particularly important for "the other half" or the other two-thirds or three-fourths who will follow Track One. In general, they possess certain identifying characteristics and there are important implications of these:

(1) They have been none too good in arithmetic. They are afraid of mathematics and dislike it. They require much patience and careful handling.

(2) They do not grasp abstractions readily. They need much concreteness, visual education of all sorts, application problems, drill and much activity on their own part.

(3) They acquire their vocabulary slowly and with difficulty. Careful definition and explanation and exemplification of all new terms is called for.

(4) They need as far as possible to develop their own rules, or at least to assist the instructor in developing them inductively wherever possible, and to have those rules made meaningful immediately by clear, practical applications.

(5) They respond well to personalization of problems, that is, a study of problems about certain named persons or certain families or groups continued through many pages or days of work.

(6) They especially need individualization. For them both group and individual diagnosis and appropriate remedial work is called for, likewise there is need for differentiating materials of instruction for them, providing for the slow and for the more mathematically agile as well as the middle group.

(7) Their instruction should call for frequent reviews to offset the ravages of quick forgetting.

(8) Even more than the brighter group they need to be taught independence and the habit of checking results.

(9) The problem of their motivation is unusually important.

Care must be taken to avoid appeals to fear or worry or over-emphasis on marks or grades. Rather emphasis must be placed upon achievement and experiencing the satisfactions of being able to do things.

(10) They must especially be shown the values of and be given insight into uses of what they learn. There should be much application, much socialization for those of Track One.

CONCLUSION

I am not pessimistic about what the next few years will bring. Ten years ago I was much discouraged. Today I see the signs of awakening among the people who come to our summer schools, through the groups that meet in schools and teachers associations throughout the country through the year, and from the bookmen who meet mathematics teachers. The movement in the general direction of the trends that I described is gathering momentum. True, the application of these ideas in the schools is developing more rapidly in some sections than in others, but it is a nationwide movement. I think a great day is coming—a day like that in which I was in high school, when 80 to 90% of all students will be studying some form of mathematics. The question does not seem to me to be "if" but "how soon" and "who will be the leaders."

FLYING LABORATORY TESTS

A "flying laboratory" equipped with "tomorrow's radio and radar devices" has cut test hours by more than fifty per cent here at the Bendix Radio division of Bendix Aviation Corporation, it was disclosed today by W. L. Webb, director of fesearch and engineering for the company, world's largest developer and manufacturer of aircraft radio, and a top producer of radar and other precision communications and navigation equipment.

The specially equipped plane, a Lockheed "Lodestar" takes the air almost daily to test performance and airworthiness of such devices as automatic radio compasses, instrument landing systems and direction finders, and other new developments, Webb stated.

The airborne research laboratory acts as a double check on performance data obtained from simulated "life tests" with elaborate laboratory equipment and enables the division to explore new developments and verify their operating characteristics with a minimum of delays.

Flight research operations of the division are in charge of Ruel Colvin. Colvin, formerly a Design Engineer for Radio Research, predecessor company of Bendix Radio, pioneered first flight tests of the radio compass, now standard on military aircraft and world airlines. He is assisted by George Bevins, flight research engineer, formerly an instructor for Eastern Air Lines and in charge of flight research activities for the Sperry Gyroscope Company.